**Vertical Spring**

**補充例題 201-1**
Show that a block hanging from a vertical spring, as in the figure, executes simple harmonic motion.

**Solution:**

The block is subject to two forces: the upward force exerted by the spring and the downward force due to gravity. If $x_0$ is the equilibrium extension at which these two forces balance, then

$$mg = kx_0$$

For any extension $x$, the net force on the block is

$$F = mg - kx = -k(x - x_0) = -kx'$$

where $x' = x - x_0$ is the displacement from the equilibrium position. Since the restoring force is linearly proportional to the displacement from equilibrium, the motion will be simple harmonic motion.

**補充習題 201-1**

A spring and a block are arranged to oscillate (1) on a frictionless horizontal surface as in Fig. 16-5, (2) on a frictionless slope of $45^\circ$ (with the block at the lower end of the spring), and (3) vertically while hanging from a ceiling. Rank the arrangements according to (a) the rest length of the spring and (b) the frequency of oscillation, greatest first.

**補充習題 201-2**

There springs each hang from a ceiling with a stationary block attached to the lower end. The blocks (with $m_1 > m_2 > m_3$) stretch the springs by equal distances. Each block-spring system is then set into vertical SHM. Rank the masses according to the period of oscillation, greatest first.

**補充習題 201-3**

A 20N weight is hung from the bottom of a vertical spring, causing the spring to stretch 20 cm. (a) What is the spring constant? (b) This spring is now placed horizontally on a frictionless table. One weight is then moved (stretching the spring) and released from rest. What is the period of oscillation?

**補充習題 201-4**

A 50.0 g mass is attached to the bottom of a vertical spring and set vibrating. If the maximum speed of the mass is 15.0 cm/s and the period is 0.500 s, find (a) the constant of the spring, (b) the amplitude of the motion, and (c) the frequency of oscillation.

**補充習題 201-5**

A vertical spring stretches 9.6 cm when a 1.3 kg block is hung from its end. (a) Calculate the spring constant. This block is then distanced an additional 5.0 cm downward and released from rest. Find (b) the period, (c) the frequency, (d) the
amplitude, and (e) the maximum speed of the resulting SHM.

補充習題 201-6
A massless spring with spring constant 19 N/m hangs vertically. A body of mass 0.20 kg is attached to its free end and then released. Assume that the spring was unstretched before the body was released. Find (a) how far below the initial position the body descends, and (b) the frequency and (c) the amplitude of the resulting motion, assumed to be simple harmonic.

補充習題 201-7
Two oscillating systems that you have studied are the block-spring and the simple pendulum. You can prove an interesting relation between them. Suppose that you hang a weight on the end of a spring, and when the weight is at rest, the spring is stretched a distance \( h \) as in the figure. Show that the frequency of this block-spring system is the same as that of a simple pendulum whose length is \( h \).

Pendulum in the noninertial frame

補充習題 201-8
A pendulum suspended from the ceiling of an elevator cab has period \( T \) when the cab is stationary. Is the period larger, smaller, or the same when the cab moves (a) upward with constant speed, (b) upward with constant speed, (c) downward with constant upward acceleration, (d) upward with constant upward acceleration, (e) upward with constant downward acceleration \( a = g \), and (f) downward with constant downward acceleration \( a = g \)?

補充習題 201-9
A pendulum mounted in a cart has period \( \pi \) when the cart is stationary and on a horizontal plane. Is the period larger, smaller, or the same if the cart is on a plane inclined at angle \( \theta \) (Fig. 16-27) and (a) stationary, (b) moving down the plane with constant speed, (c) moving up the plane with constant speed, (d) moving up the plane with constant acceleration directed up the plane, (e) moving down the plane with constant acceleration directed up the plane, (f) moving down the plane with acceleration \( a = g \sin \theta \) directed down the plane, and (g) moving up the plane with acceleration \( a = g \sin \theta \) directed down the plane?
Physical Pendulum

補充習題 201-10 打棒球：美妙的點
The center of oscillation of a physical pendulum has this interesting property: if an impulsive force (assumed horizontal and in the plane of oscillation) acts at the center of oscillation, no reaction is felt at the point of support. Baseball players (and players of many other sports) know that unless the ball hits the bat at this point (called the “sweet spot” by athletes), the reaction due to the impact will sting their hands. To prove this property, let the stick in Fig. 16-12a simulate a baseball bat. Suppose that a horizontal force $F$ (due to impact with the ball) acts toward the right at $P$, the center of oscillation. The batter is assumed to hold the bat at $\theta$, the point of support of the stick.
(a) What acceleration does point $\theta$ undergo as a result of $F$? (b) What angular acceleration is produced by $F$ about the center of mass of the stick? (c) As a result of the angular acceleration in (b), what linear acceleration does point $\theta$ undergo? (d) Considering the magnitudes and directions of the accelerations in (a) and (c), convince yourself that $P$ is indeed the “sweet spot”.

補充習題 201-11
A physical pendulum has two possible pivot points $A$ and $B$, point $A$ has a fixed position, and $B$ is adjustable along the length of the pendulum, as shown in the figure. The period of the pendulum when suspended from $A$ is found to be $T$. The pendulum is then reversed and suspended from $B$, which is moved until the pendulum again has period $T$. Show that the free-fall acceleration $g$ is given by

$$g = \frac{4\pi^2 L^2}{T^2}$$

in which $L$ is the distance between $A$ and $B$ for equal periods $T$. (Note that $g$ can be measured in this way without knowing the rotational inertia of the pendulum or any of its dimensions except $L$.)