Problem 22-87

Ch22, Problem 87

Find the frequency of oscillation of an electric dipole, of dipole moment \( \vec{p} \) and rotational inertia \( I \), for small amplitudes of oscillation about its equilibrium position in a uniform electric field of magnitude \( E \).

Solution:

The magnitude of the torque acting on the dipole is given by \( \tau = \vec{p}E \sin \theta \), where \( \vec{p} \) is the magnitude of the dipole moment, \( E \) is the magnitude of the electric field, and \( \theta \) is the angle between the dipole moment and the field. It is a restoring torque: it always tends to rotate the dipole moment toward the direction of the electric field. If \( \theta \) is positive, the torque is negative and vice versa.

Write \( \tau = -\vec{p}E \sin \theta \). If the amplitude of the motion is small, we may replace \( \sin \theta \) with \( \theta \) in radians.

Thus, \( \tau = -\vec{p}E \theta \). Since the magnitude of the torque is proportional to the angle of rotation, the dipole oscillates in simple harmonic motion, just like a torsional pendulum with torsion constant \( \kappa = \vec{p}E \). The angular frequency \( \omega \) is given by

\[
\omega^2 = \frac{\kappa}{I} = \frac{\vec{p}E}{I},
\]

and the frequency of oscillation is

\[
f = \frac{\omega}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{\vec{p}E}{I}}.
\]